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# LARGE DEFLECTION OF RECTANGULAR PLATE WITH HIGH ASPECT RATIO

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# Large Deflection of Rectangular Plate With High Aspect Ratio

by

Cheng Chih Shao

Presented as partial fulfillment of the requirements for the degree of
Master of Engineering in Naval Architecture.

University of California, Berkeley, California

January, 1964

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# NOTATION

| | |
|---|---|
| a | plate length in x-direction |
| b | plate length in y-direction |
| h | plate thickness |
| p | lateral pressure |
| $\omega$ | Vertical displacement of points of the middle surface |
| $\sigma_x'$ , $\sigma_y'$ | membrane stress parallel to x, and y axes |
| $\tau_{xy}'$ | membrane shearing stress |
| $\sigma_x''$ , $\sigma_y''$ | extreme-fiber bending and shearing stress parallel to x and y axes |
| $\tau_{xy}''$ | bending shearing stress |
| $m_x$ , $m_y$ | bending moments per unit length of sections of a plate perpendicular to x and y axes, respectively |
| $\epsilon_x'$ , $\epsilon_y'$ | unit elongations in x and y direction |
| $\gamma_{xy}'$ | shearing strain component |
| E | modulus of elasticity in tension and compression |
| $\mu$ | poisson's ratio |
| D | flexure rigidity of the plate |
| F | stress-function |

Subscripts m, n, m', n', r, s, p, q, k, t and $\ell$ represent integers.

## 1. Introduction

Plates, generally speaking, deflect as nondevelopable surface. As a consequence, the increase of deflections beyond a certain magnitude is accompanied by the so-called membrane stresses acting in the plane of the plate. In order to describe the behavior of the plate at deflections equal to or exceeding its thickness but still small compared to its lateral dimensions, Von Karman derived the large-deflection theory of isotropic plate by assuming that the external loads are resisted by both bending action and forces in the middle plane of the plate.

Von Karman's fundamental equations for large-deflection theory has been solved by many authors, but most of them are of an approximate nature. The only theoretically exact solutions were given by S. Levy. But he gave the results for square plate only since his theoretically exact solutions are extremely cumbersome. So, up to the time being, we still do not know exactly the behaviors of a rectangular plate with high aspect ratio at large deflection. As it is well known in designs of thin plates that bend under lateral loading, formulas based on small deflection theory which neglects stretching and shearing in the middle surface may still be used with sufficient accuracy for deflections up to fifty per cent of the plate thickness. This is true for a square plate or a rectangular plate with low aspect ratio. If a rectangular plate with high aspect ratio is being considered, we cannot make sure whether it is true or not, since no exact results have been given for such a rectangular plate at large deflection. It is the purpose of this paper to get an insight into the behaviors of

plates with high aspect ratio by means of S. Levy's theoretical exact method.

It should be mentioned here that this paper has been restricted as to:

a) all edges simply supported
b) lateral uniform pressure, total edge load zero
c) aspect ratio $\beta = 5$

This shows that the areas for further research have been by no means exhausted.

The choosing of aspect ratio $\beta = 5$ is not arbitrary, since this value is the upper limitation in the ship structure. Moreover, beyond this value, the behavior of the plate is nearly as a strip. And it is, of course, out of the purpose of this paper if the value of the aspect ratio is small.

## 2. Fundamental equations for the deformation of thin plates

Consider a rectangular isotropic elastic plate having the thickness $h$ and rigidity $D$. Let the origin of a cartesian coordinate system $x, y, z$ be located at a corner of the middle plane of the plate, the axes $x$ and $y$ being directed along the edges of the plate of length $a$ and $b$, respectively. The plate is acted on by an arbitrary lateral load $p(x, y)$, which provokes a displacement field in the plate characterized by the components $u, v, w,$ the last one representing the deflection of the plate. The fundamental equations governing the deflection of thin plates were developed by Von Karman. They were given by Timoshenko [3]* in essentially the following forms:

---

*Numbers in the brackets, [ ], refer to references at the end of this paper.

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{1}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{h}{D}\left[\frac{p}{h} + \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\right] \tag{2}$$

where the membrane stresses are

$$\sigma_x' = \frac{\partial^2 F}{\partial y^2} \tag{3a}$$

$$\sigma_y' = \frac{\partial^2 F}{\partial x^2} \tag{3b}$$

$$\tau_{xy}' = -\frac{\partial^2 F}{\partial x \partial y} \tag{3c}$$

the median-fiber strains are

$$\epsilon_x' = \frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2} - \mu\frac{\partial^2 F}{\partial x^2}\right) \tag{4a}$$

$$\epsilon_y' = \frac{1}{E}\left(\frac{\partial^2 F}{\partial x^2} - \mu\frac{\partial^2 F}{\partial y^2}\right) \tag{4b}$$

$$\gamma_{xy}' = -\frac{2(1+\mu)}{E}\frac{\partial^2 F}{\partial x \partial y} \tag{4c}$$

and the extreme-fiber bending and shearing stresses are

$$\sigma_x'' = -\frac{E h}{2(1-\mu^2)}\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) \tag{5a}$$

$$\sigma_y'' = -\frac{Eh}{2(1-\mu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) \tag{5b}$$

$$\tau_{xy}'' = -\frac{Eh}{2(1+\mu)}\frac{\partial^2 w}{\partial x \partial y} \tag{5c}$$

3. General solution for simply supported rectangular plate

In case of simply supported rectangular plate, the boundary conditions referred to deflection and moment are

$$w=0,\ M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) = 0 \quad \text{when } x = 0,\ a \tag{6a}$$

$$w=0,\ M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) = 0 \quad \text{when } y = 0,\ b \tag{6b}$$

One of the boundary conditions referred to the stress function F is that there are no shearing stresses along the edges. This boundary condition can be written as

$$\frac{\partial^2 F}{\partial x \partial y} = 0 \qquad \text{at the boundaries;} \tag{7}$$

another is that the boundaries remain straight but may displace perpendicular to their lengths. It should be noted that, if this is done, the distribution of the in-plane stresses perpendicular to the boundaries cannot be specified, and we can merely state that the integral of the in-plane stress along the boundary must be equal to the total applied load along the boundary.

An exact solution of this problem can be established by starting from the simultaneous equations (1) and (2).

The deflection of the plate may be taken in the Navier form

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{m,n} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{8}$$

the boundary conditions (6) thus being satisfied by any, yet unknown, values of the coefficients $w_{m,n}$. The given lateral load may be expanded in a double Fourier series

$$p = \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} p_{r,s} \sin \frac{r\pi x}{a} \sin \frac{s\pi y}{b} \tag{9}$$

A suitable expression for the Airy stress function, then, is

$$F = \frac{\bar{P}_x y^2}{2bh} + \frac{\bar{P}_y x^2}{2ah} + \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} f_{p,q} \cos \frac{p\pi x}{a} \cos \frac{q\pi y}{b} \tag{10}$$

where

$$\bar{P}_x = h \int_0^b \sigma_x' \, dy \tag{11a}$$

$$\bar{P}_y = h \int_0^a \sigma_y' \, dx \tag{11b}$$

are constants denoting the total tension load applied on the sides $x = o$, $a$ and $y = o$, $b$, respectively. Substituting the expressions (8) and (10) into (1) we get the following relation between the coefficients of both series:

$$\sum_{p=0,1}^{\infty} \sum_{q=0,1}^{\infty} \left(p^2 \frac{b}{a} + q^2 \frac{a}{b}\right)^2 f_{p,q} \cos \frac{p\pi x}{a} \cos \frac{q\pi y}{b}$$

$$= E \sum_{m=1}^{\infty} \sum_{m'=1}^{\infty} \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} w_{m,n} w_{m',n'} \, m n$$

$$\Big[\left(\frac{m'n'-mn}{4}\right)\cos(m+m')\frac{\pi x}{a}\cos(n+n')\frac{\pi y}{b}$$
$$+\left(\frac{m'n'+mn}{4}\right)\cos(m+m')\frac{\pi x}{a}\cos(n-n')\frac{\pi y}{b}$$
$$+\left(\frac{m'n'+mn}{4}\right)\cos(m-m')\frac{\pi x}{a}\cos(n+n')\frac{\pi y}{b}$$
$$+\left(\frac{m'n'-mn}{4}\right)\cos(m-m')\frac{\pi x}{a}\cos(n-n')\frac{\pi y}{b}\Big] \qquad (12)$$

the coefficients in stress function, $f_{p,q}$, are to be determined by the values of the coefficients $w_{m,n}$. The sum includes all products for which $m \pm m' = \pm p$ and $n \pm n' = \pm q$.

Example: Take a square plate ($a = b$), and consider the first six terms only (i.e., $w_{1,1}$, $w_{1,3}$, $w_{1,5}$, $w_{3,1}$, $w_{5,1}$ and $w_{3,3}$). For $f_{0,2}$ ($p = 0$, $q = 2$), the following nine groups should be considered:

$$\begin{bmatrix} m=1 & n=1 \\ m'=1 & n'=1 \end{bmatrix} \quad \begin{bmatrix} m=1 & n=3 \\ m'=1 & n'=1 \end{bmatrix} \quad \begin{bmatrix} m=1 & n=1 \\ m'=1 & n'=3 \end{bmatrix}$$

$$\begin{bmatrix} m=1 & n=5 \\ m'=1 & n'=3 \end{bmatrix} \quad \begin{bmatrix} m=1 & n=3 \\ m'=1 & n'=5 \end{bmatrix} \quad \begin{bmatrix} m=3 & n=1 \\ m'=3 & n'=1 \end{bmatrix}$$

$$\begin{bmatrix} m=3 & n=3 \\ m'=3 & n'=1 \end{bmatrix} \quad \begin{bmatrix} m=3 & n=1 \\ m'=3 & n'=3 \end{bmatrix} \quad \begin{bmatrix} m=5 & n=1 \\ m'=5 & n'=1 \end{bmatrix}$$

and we obtain:

$$f_{0,2} = \frac{E}{32}\Big[(\omega_{1,1})^2 - 2\,\omega_{1,1}\,\omega_{1,3} + 9(\omega_{3,1})^2 - 18\,\omega_{3,1}\,\omega_{3,3}$$
$$- 2\,\omega_{1,3}\,\omega_{1,5} + 25(\omega_{5,1})^2\Big]$$

Inserting the expressions (8), (9) and (10) into equation (2), we can establish the relation between the deflections, the stress functions and the lateral pressure. The general equation given by Levy[1] is as follows:

$$p_{r,s} = D\,\omega_{r,s}\,\pi^4\left(\frac{r^2}{a^2}+\frac{s^2}{b^2}\right)^2 + \bar{p}_x\,\omega_{r,s}\,\pi^2\frac{r^2}{ba^2} + \bar{p}_y\,\omega_{r,s}\,\pi^2\frac{s^2}{ab^2}$$
$$+\frac{h\pi^4}{4a^2b^2}\Big\{-\sum_{k=1}^{r}\sum_{t=1}^{s}\left[(s-t)k-(r-k)t\right]^2 f_{(r-k),(s-t)}\,\omega_{k,t}$$

$$
\begin{aligned}
&-\sum_{k=0}^{\infty}\sum_{t=0}^{\infty}\left[t(k+r)-k(t+s)\right]^2 f_{k,t}\,\omega_{(k+r),(t+s)}\\
&+\sum_{k=0}^{\infty}\sum_{t=1}^{\infty}\left[(k+r)(t+s)-kt\right]^2 f_{k,(t+s)}\,\omega_{(k+r),t}\\
&+\sum_{k=1}^{\infty}\sum_{t=0}^{\infty}\left[tk-(k+r)(t+s)\right]^2 f_{(k+r),t}\,\omega_{k,(t+s)}\\
&-\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\left[(t+s)k-(k+r)t\right]^2 f_{(k+r),(t+s)}\,\omega_{k,t}\\
&-\sum_{k=1}^{r}\sum_{t=0}^{\infty}\left[tk+(r-k)(t+s)\right]^2 f_{(r-k),t}\,\omega_{k,(t+s)}\\
&+\sum_{k=1}^{r}\sum_{t=1}^{\infty}\left[(t+s)k+(r-k)t\right]^2 f_{(r-k),(t+s)}\,\omega_{k,t}\\
&-\sum_{k=0}^{\infty}\sum_{t=1}^{s}\left[(s-t)(k+r)+tk\right]^2 f_{k,(s-t)}\,\omega_{(k+r),t}\\
&+\sum_{k=1}^{\infty}\sum_{t=1}^{s}\left[(s-t)k+t(k+r)\right]^2 f_{(k+r),(s-t)}\,\omega_{k,t}\Bigg\}
\end{aligned}
\tag{13}
$$

## 4. Specific solution for rectangular plate with high aspect ratio under uniform lateral pressure, total edge load zero

a. General: In this special case, the total edge load in the x-direction $\bar{P}_x$ and in the y-direction $\bar{P}_y$ is zero, and the uniform lateral pressure is constant. The expansion of this pressure $p$ in a Fourier series as shown in equation (9) gives pressure coefficients $p_{r,s}=\frac{1}{rs}\left(\frac{4}{\pi}\right)^2 p$. In the solution of a rectangular plate with such a high aspect ratio as $\beta = 5$, we assume that, in the deflection coefficients $w_{m,n}$, $m = 1$ and $n = 15$ are accuracy enough. So we consider $w_{1,1}$, $w_{1,3}$, $w_{1,5}$, $w_{1,7}$, $w_{1,9}$, $w_{1,11}$, $w_{1,13}$ and $w_{1,15}$ only. All other terms assumed to be equal to zero. Under this assumption, a general equation for the coefficients in the stress-function can be obtained from equation (12) as follows:

$$f_{p,q} = C_{p,q} \frac{\beta^2 E}{4(p^2\beta^2+q^2)^2} \tag{14}$$

where

$$C_{0,0} = 0$$

$$C_{2,0} = 2 \sum_{l=1,3}^{15} l^2 (\omega_{1,l})^2$$

as $2 \leq q \leq 14$

$$C_{0,q} = q \sum_{l=1,3}^{q-1} (q-l)\, \omega_{1,l}\, \omega_{1,(q-l)} - q^2 \sum_{l=1,3}^{15-q} \omega_{1,l}\, \omega_{1,(q+l)}$$

$$C_{2,q} = \sum_{l=1,3}^{q-1} (2l-q)(q-l)\, \omega_{1,l}\, \omega_{1,(q-l)} + \sum_{l=1,3}^{15-q} (2l+q)^2 \omega_{1,l}\, \omega_{1,(l+q)}$$

as $16 \leq q \leq 30$

$$C_{0,q} = q \sum_{l=q-15}^{15} (q-l)\, \omega_{1,l}\, \omega_{1,(q-l)}$$

$$C_{2,q} = \sum_{l=q-15}^{15} (2l-q)(q-l)\, \omega_{1,l}\, \omega_{1,(q-l)}$$

Example

$$f_{2,8} = \frac{\beta^2 E}{2(\beta^2+16)} (-9\omega_{1,1}\omega_{1,7} - \omega_{1,3}\omega_{1,5} + 25\omega_{1,1}\omega_{1,9}$$

$$+ 49\omega_{1,3}\omega_{1,11} + 81\omega_{1,5}\omega_{1,3} + 121\omega_{1,7}\omega_{1,15})$$

Equation (13) represents a doubly infinite family of equations. In this special case, equation (13) can be reduced as

$$-\frac{64}{\pi^6}\frac{\beta^2}{s}\left(\frac{pa^4}{E h^4}\right) + \frac{1}{3(1-\mu^2)} \frac{(\beta^2+s^2)^2}{h\beta^2} \omega_{1,s} + \frac{1}{h^3 E}\Big[-2\sum_{t=1,3}^{s} (s-t)^2 f_{0,(s-t)}\, \omega_{1,t}$$

$$-2\sum_{t=0,2}^{15-s} t^2 f_{0,t}\, \omega_{1,(t+s)} + 2\sum_{t=1,3}^{15} (t+s)^2 f_{0,(t+s)}\, \omega_{1,t}$$

$$+\sum_{t=0,2}^{15-s}(t+2s)^2 f_{2,t}\,\omega_{1,(t+s)} - \sum_{t=1,3}^{15}(s-t)^2 f_{2,(t+s)}\,\omega_{1,t}$$

$$+\sum_{t=1,3}^{s}(s+t)^2 f_{2,(s-t)}\,\omega_{1,t}\Big] = 0 \qquad (15)$$

where S = 1, 3, 5, ---- 15, representing the first eight simultaneous equations of the family.

Example: S = 9

$$-\frac{64}{\pi^6}\frac{\beta^2}{9}\left(\frac{pa^4}{Eh^4}\right)+\frac{1}{3(1-\mu^2)}\frac{(\beta^2+81)^2}{h\beta^2}\,\omega_{1,s}$$

$$+\frac{1}{h^3E}\Big[4(-32f_{0,8}+50f_{0,10}+25f_{2,8}-16f_{2,10})\,\omega_{1,1}$$

$$+4(-18f_{0,6}+72f_{0,12}+36f_{2,6}-9f_{2,12})\,\omega_{1,3}$$

$$+4(-8f_{0,4}+98f_{0,14}+49f_{2,4}-4f_{2,14})\,\omega_{1,5}$$

$$+4(-2f_{0,2}+128f_{0,16}+64f_{2,2}-f_{2,16})\,\omega_{1,7}$$

$$+648(f_{0,18}+f_{2,0})\,\omega_{1,9}$$

$$+4(-2f_{0,2}+200f_{0,20}+100f_{2,2}-f_{2,20})\,\omega_{1,11}$$

$$+4(-8f_{0,4}+242f_{0,22}+121f_{2,4}-4f_{2,22})\,\omega_{1,13}$$

$$+4(-18f_{0,6}+288f_{0,24}+144f_{2,6}-9f_{2,24})\,\omega_{1,15}\Big]=0$$

In each of the equations of the family the coefficients for the stress function, $f_{p,q}$, may be replaced by their values as obtained from equation (14). The resulting equations will involve the lateral pressure p and the cubes of deflection coefficients $w_{m,n}$. Solving these eight simultaneous nonlinear equations, we obtain the values of the eight deflection coefficients, $w_{1,1}$, $w_{1,3}$ ---- $w_{1,15}$. The results are given in Table (1) and shown in Fig. (1, 2). The straight lines give the corresponding solution by the linear theory [3].

The equation of the deflection profile for the center line parallel to the long sides of the plate can be written as:

$$w = \sum_{n=1,3}^{15} w_{1,n} \cos \frac{n\pi y}{b} \tag{16}$$

a part of the results are given in Table (2), and three curves are shown in Fig. (3).

b. Membrane stresses: From equations (3) and (10) we can derive the equations for the membrane stresses as follows:

(a) $\sigma_x'$ along $y = 0, b$

$$\left(\frac{\sigma_x' a^2}{E h^2}\right)_{y=0,b} = -\frac{\pi^2}{E h^2 \beta^2} \sum_{q=0,2}^{30} q^2 \left(f_{0,q} + f_{2,q} \cos \frac{2\pi x}{a}\right) \tag{17a}$$

(b) $\sigma_x'$ along $x = 0, a$

$$\left(\frac{\sigma_x' a^2}{E h^2}\right)_{x=0,a} = -\frac{\pi^2}{E h^2 \beta^2} \sum_{q=0,2}^{30} q^2 \left(f_{0,q} + f_{2,q}\right) \cos \frac{q\pi y}{b} \tag{17b}$$

(c) $\sigma_x'$ along $x = \frac{a}{2}$

$$\left(\frac{\sigma_x' a^2}{E h^2}\right)_{x=\frac{a}{2}} = -\frac{\pi^2}{E h^2 \beta^2} \sum_{q=0,2}^{30} q^2 \left(f_{0,q} - f_{2,q}\right) \cos \frac{q\pi y}{b} \tag{17c}$$

(d) $\sigma_y'$ along $y = 0, b$

$$\left(\frac{\sigma_y' a^2}{E h^2}\right)_{y=0,b} = -\frac{4\pi^2}{E h^2}\sum_{q=0,2}^{30} f_{2,q}\cos\frac{2\pi x}{a} \tag{17d}$$

(e) $\sigma_y'$ along $x = 0, a$

$$\left(\frac{\sigma_y' a^2}{E h^2}\right)_{x=0,a} = -\frac{4\pi^2}{E h^2}\sum_{q=0,2}^{30} f_{2,q}\cos\frac{q\pi y}{b} \tag{17e}$$

(f) $\sigma_y'$ along $x = \frac{a}{2}$

$$\left(\frac{\sigma_y' a^2}{E h^2}\right)_{x=\frac{a}{2}} = +\frac{4\pi^2}{E h^2}\sum_{q=0,2}^{30} f_{2,q}\cos\frac{q\pi y}{b} \tag{17f}$$

Substituting the values of deflection coefficients $w_{m,n}$ given in Table (1) into equation (14), and then substituting equation (14) into equation (17), we obtain all the results, a part of which are given in Tables(3-7). Three of each group are shown in Figs. (4, 5), and the membrane stress curves for some specified points are shown in Fig. (8).

c. Bending stresses: From equations (5) and (8) we can derive the equations for the bending stresses as follows:

(a) $\tau_{xy}''$ along $y = 0$

$$\left(\frac{\tau_{xy}'' a^2}{E h^2}\right)_{y=0} = -\frac{\pi^2}{2(1+\mu)h\beta}\sum_{n=1,3}^{15} n\, w_{1,n}\cos\frac{\pi x}{a} \tag{18a}$$

(b) $\tau_{xy}''$ along $x = 0$

$$\left(\frac{\tau_{xy}'' a^2}{E h^2}\right)_{x=0} = -\frac{\pi^2}{2(1+\mu)h\beta}\sum_{n=1,3}^{15} n\, w_{1,n}\cos\frac{n\pi y}{b} \tag{18b}$$

(c) $\sigma_x''$ along $x = \frac{a}{2}$

$$\left(\frac{\sigma_x'' a^2}{E h^2}\right)_{x=\frac{a}{2}} = \frac{\pi^2}{2h(1-\mu^2)}\sum_{n=1,3}^{15}\left(1+\mu\frac{n^2}{\beta^2}\right) w_{1,n}\sin\frac{n\pi y}{b} \tag{18c}$$

(d) $\sigma_y''$ along $x = \frac{a}{2}$

$$\left(\frac{\sigma_y'' a^2}{E h^2}\right)_{x=\frac{a}{2}} = \frac{\pi^2}{2h(1-\mu^2)} \sum_{n=1}^{15} \left(\frac{n^2}{\beta^2}+\mu\right) w_{1,n} \sin\frac{n\pi y}{b} \tag{18d}$$

Substituting the values of deflection coefficients $w_{m,n}$ given in Table (1) into equation (18), we obtain all the results, a part of which are given in Tables (8-11). Three of each group are shown in Figs. (6-7). Bending stress curves for some specified point and maximum bending stresses curves are shown in Fig. (9).

d. Stress intensity: For determining failure of ductile materials having a pronounced yielding point stress $\sigma_{yp}$ in simple tension, octahedral failure criterion seems to be the most significant and applicable theory [2].

Define $\sigma_x = \sigma_x' + \sigma_x''$ (19a)

$$\sigma_y = \sigma_y' + \sigma_y'' \tag{19b}$$

$$\tau_{xy} = \tau_{xy}' + \tau_{xy}'' \tag{19c}$$

Octahedral failure criterion can be written as

$$\sigma_{yp} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}} \tag{20}$$

From the results we have obtained, we can see that the values of the maximum bending stresses are much more larger than that of the maximum membrane stresses, and the maximum bending stresses occur on the center line parallel to the long sides of the plate where the shearing stresses are zero. By introducing the dimensionless notation, equation (20) becomes

$$\frac{\sigma_{yp} a^2}{E h^2} = \sqrt{\left(\frac{\sigma_x a^2}{E h^2}\right)^2 + \left(\frac{\sigma_y a^2}{E h^2}\right)^2 - \left(\frac{\sigma_x a^2}{E h^2}\right)\left(\frac{\sigma_y a^2}{E h^2}\right)} \tag{21}$$

The results are shown in Fig. (10).

## 5. Conclusion and discussion

a. From curve (2) we can see that, for design purpose, even in the case of rectangular plate with high aspect ratio, the linear theory may still be used provided that the deflections at center are small compared with the thickness. But it should be noted that the error in deflection is about 15 per cent when the deflection at center is equal to 50 per cent of the plate thickness. Comparing with the results of square plate, we can see that the error is increased as the aspect ratio of the plate is increased.

b. It is interesting to point out here that, as shown in Fig. (3), for a rectangular plate with high aspect ratio, the maximum deflection shows up at the center of the plate when the deflection is small but not at the center when the deflection is large. This is probably due to the high compressive membrane stresses $\sigma_y'$ occurring near the corners of the plate. It is evident that this compressive membrane stress is the result of the boundary condition that the edges remain straight but may displace perpendicular to their length.

c. As we can see from Fig. (8) that the membrane stresses are negligible provided that the deflections are small compared with the plate thickness, and become more and more significant as the deflections of the plate increase, and that the membrane stresses in the x-direction are always larger than that in the y-direction. It is now clear that the error in deflection generated by using small deflection theory, up to 15 per cent for deflection equal to 50 per cent of plate thickness, is mainly due to the

neglecting of membrane action in x-direction.

d. The fact that the absolute value of the membrane stresses in y-direction at the corners of the plate are just equal to that at the center of the short sides of the plate is, of course, due to the one term approximation in that direction.

e. From Fig. (9) we can see that the maximum bending stresses in the x-direction are prominent when the lateral load, as well as deflection is small, and the maximum bending stresses in the y-direction become prominent when the lateral load is large. This is due to the deflection hump occurring as the lateral load is large enough.

f. The distribution of all the stresses along the y-direction changes very rapidly near the ends of the plate and is nearly equal to constant in the middle part, about 60 per cent of the plate length. This indicates that the middle part of the plate behaves like the case of cylindrical bending.

g. Fig. (10) shows that the lateral load at which the yield stresses occur is dependent upon the yielding point stress of the material and the width-thickness ratio $(\frac{a}{h})$. Once we know the load, we can find the deflection from Fig. (2). Example:

given $E = 30 \times 10^6$ psi

$\sigma_{yp} = 39000$ psi

$a/h = 100$

then we get

$$\frac{\sigma_{yp} a^2}{E h^2} = 13 \quad , \quad \frac{p a^4}{E h^4} = 15$$

$$w_{center} = 1.425 h, \quad w_{max} = 1.5 h$$

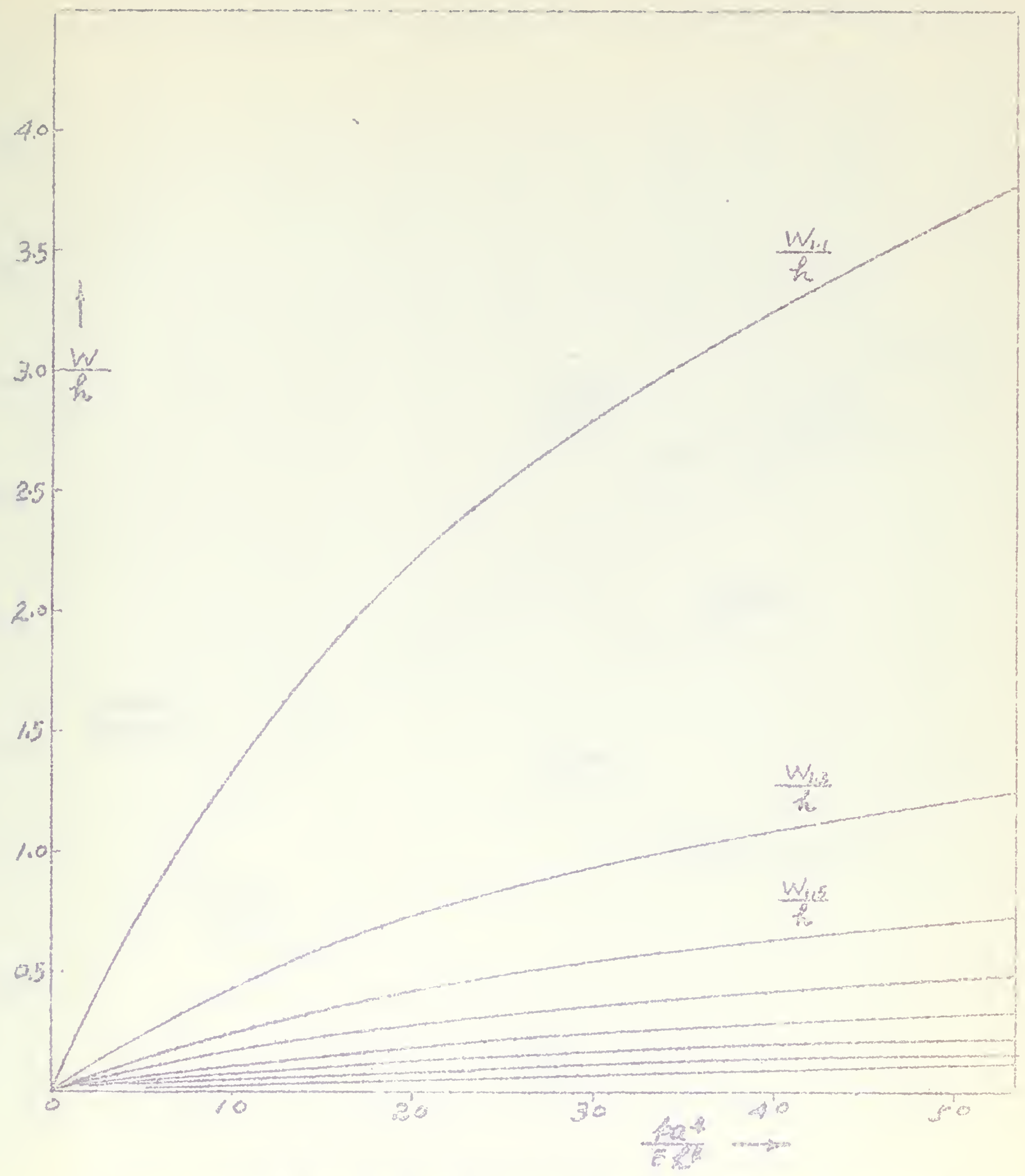


Fig. 1.-- Values of coefficients for deflection function

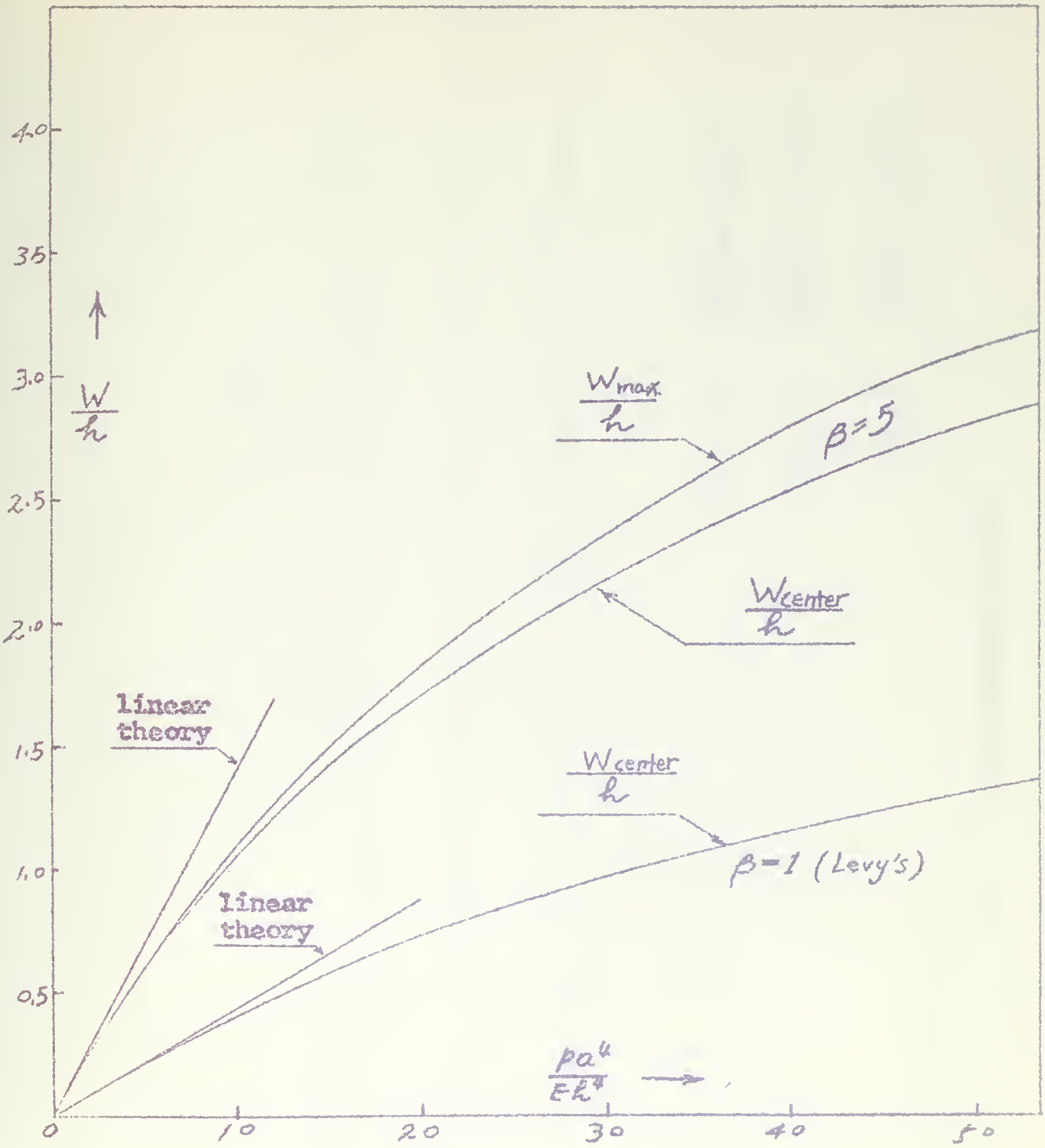


Fig. 2.-- Deflection curves

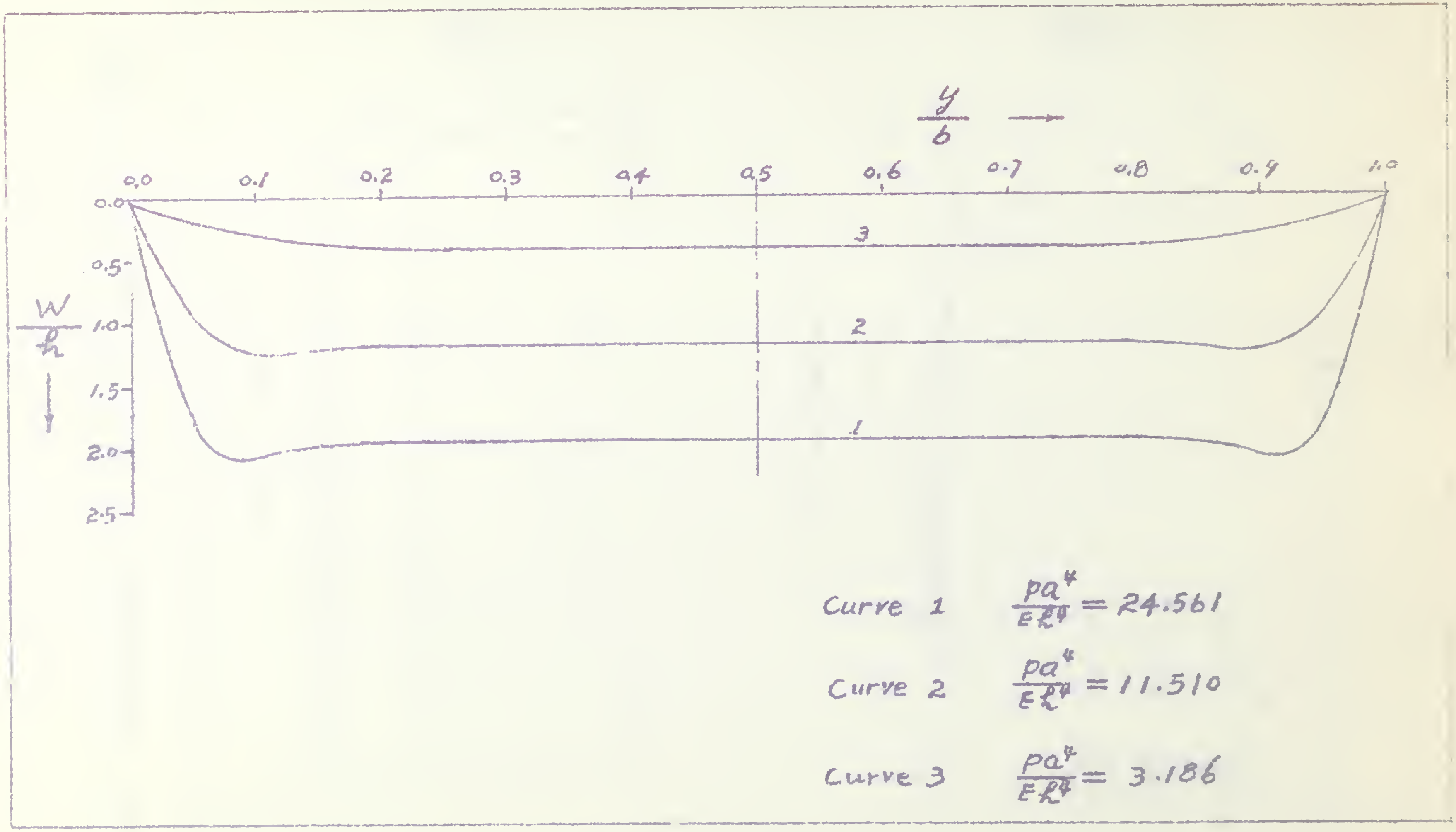
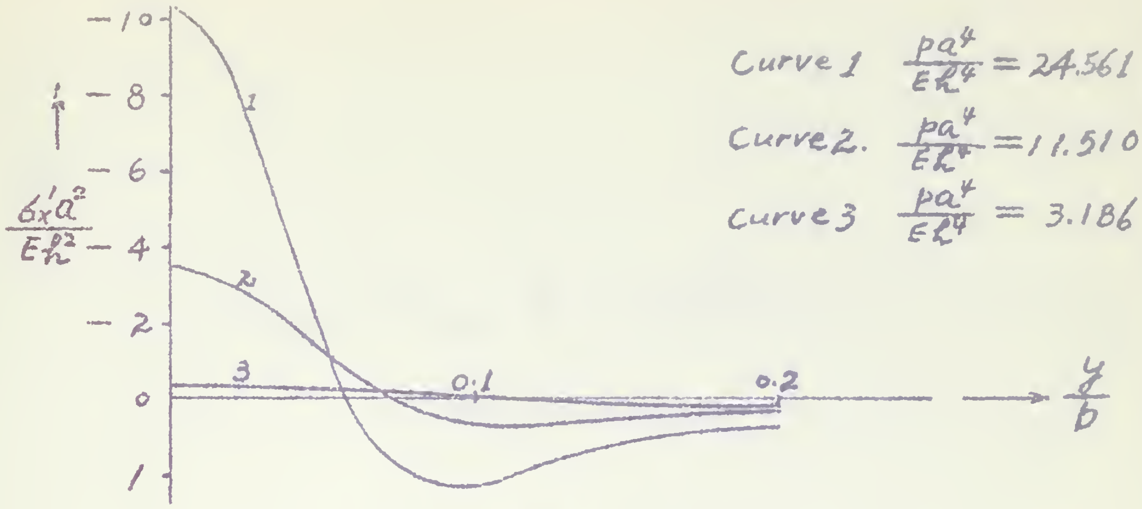
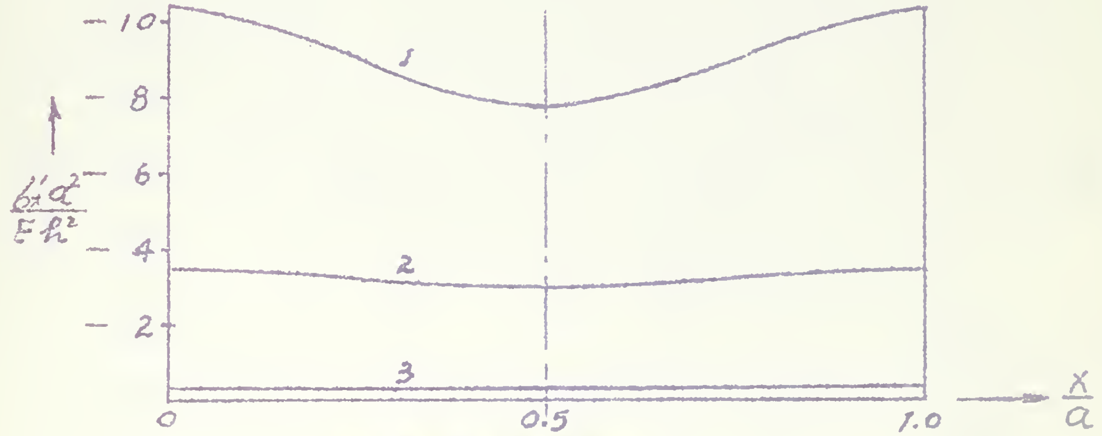


Fig. 3.-- Deflection profiles

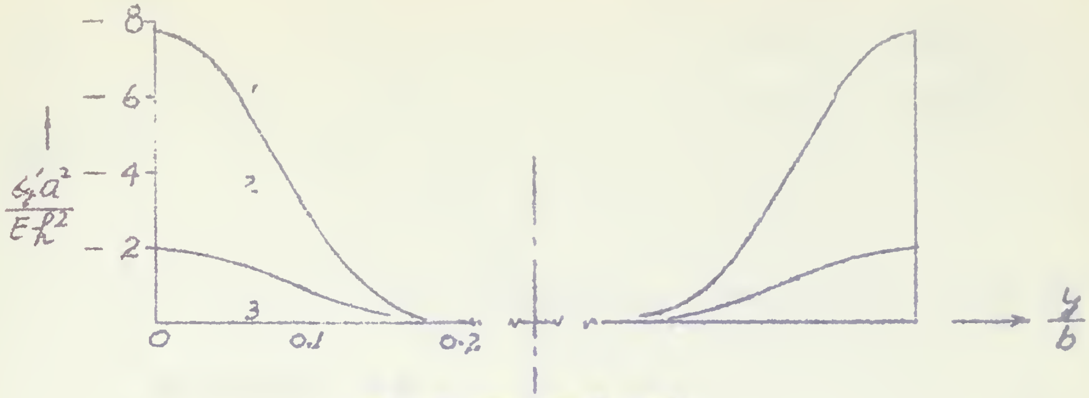


a. along x= 0,a

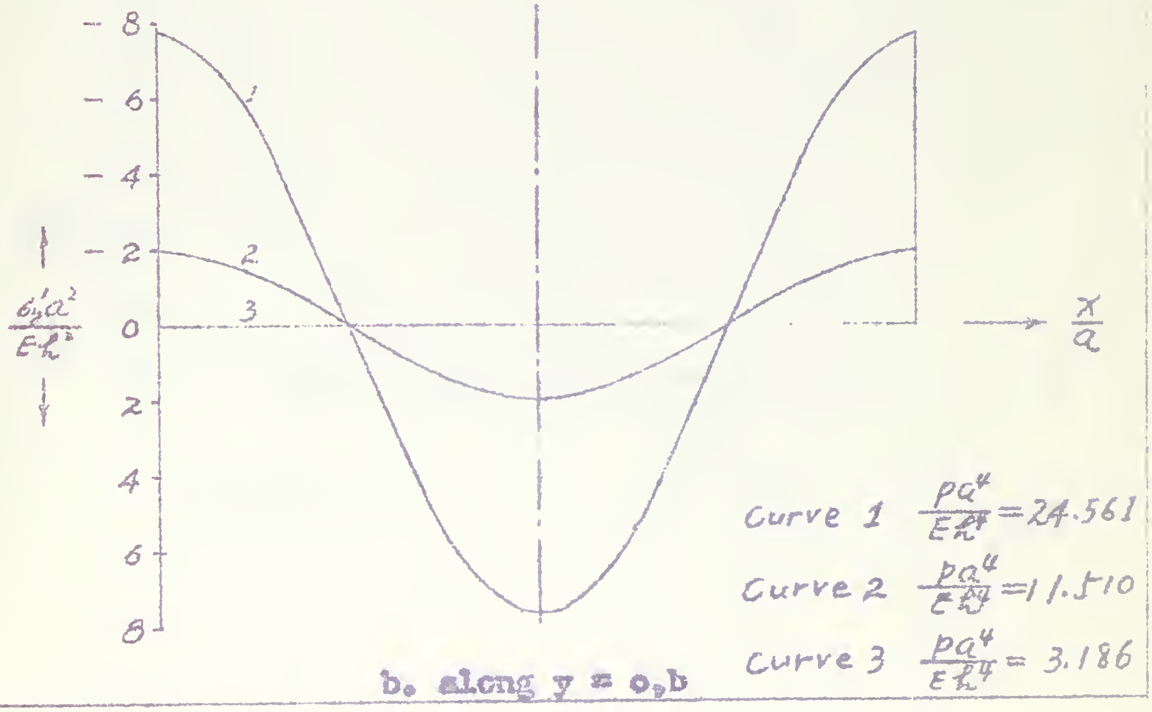


b. along y = 0,b

Fig. 4.— membrane stresses, $\sigma_x'$, distribution curve



a. along x = o,a



b. along y = o,b

Fig. 5.— Membrane stresses, $\sigma_y'$, distribution curve

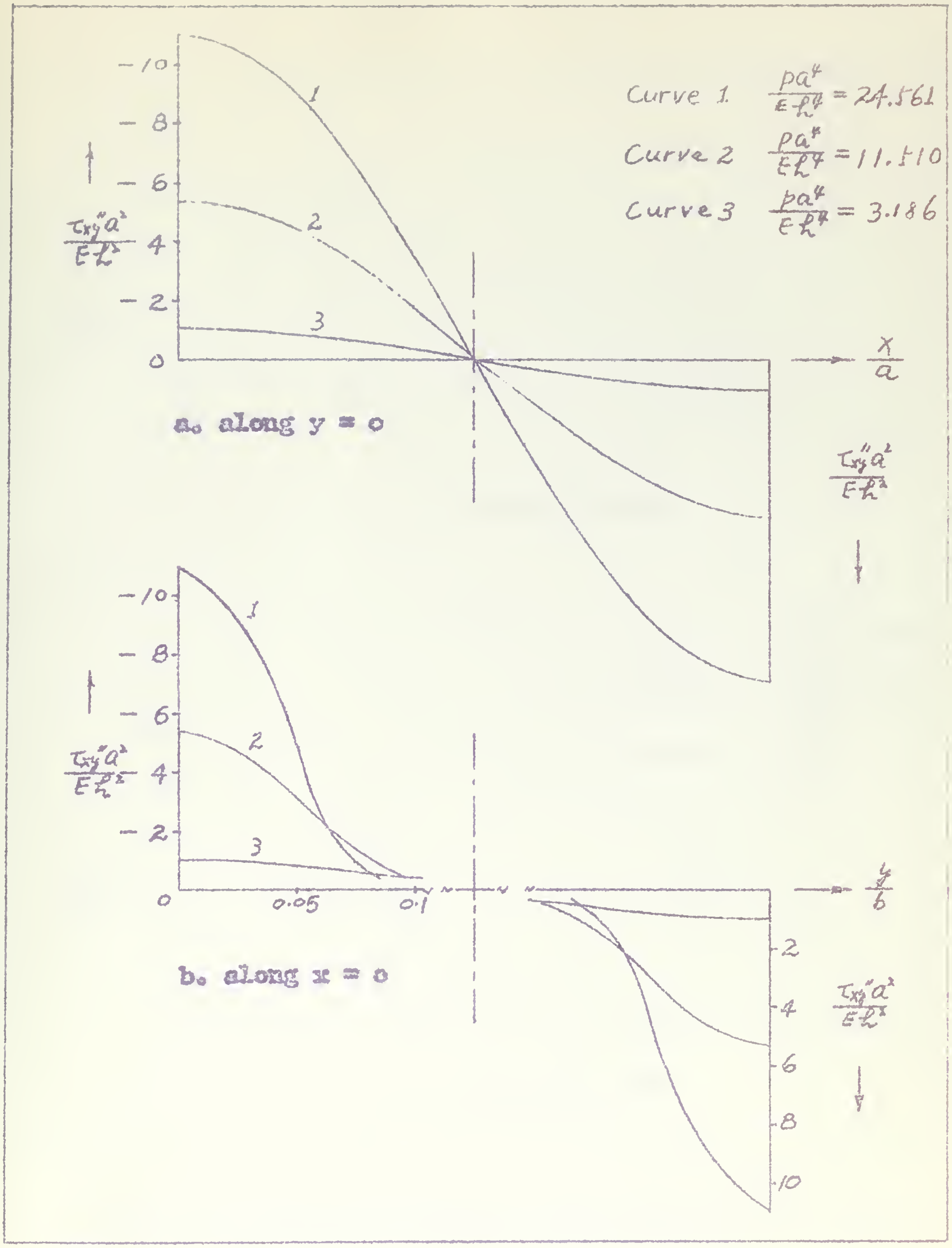
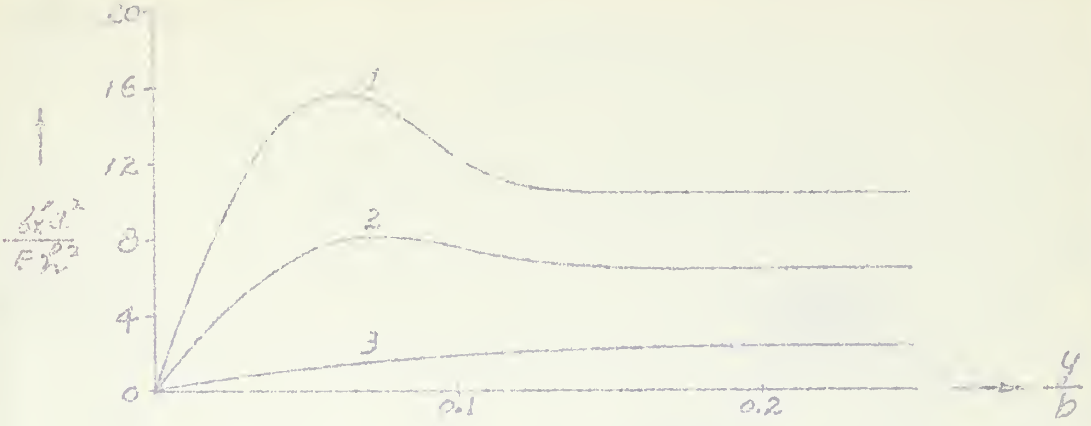
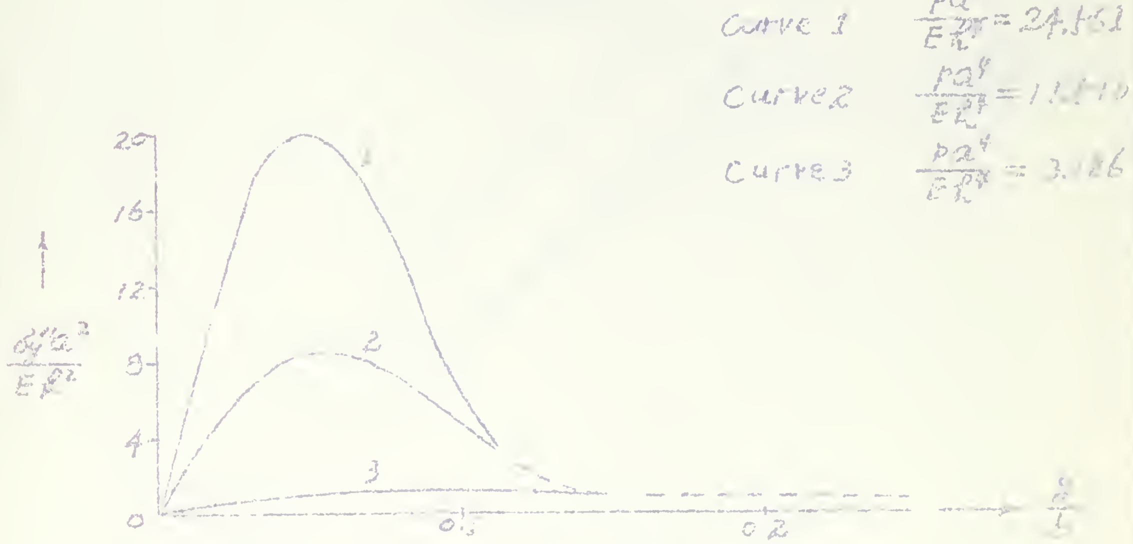


Fig. 6.-- Shearing stresses, $\tau''_{xy}$, distribution curve



a. $\sigma_x''$ along x = a/2



b. $\sigma_y''$ along x = a/2

Fig. 7.— Bending stresses, $\sigma_x''$ & $\sigma_y''$ distribution curve

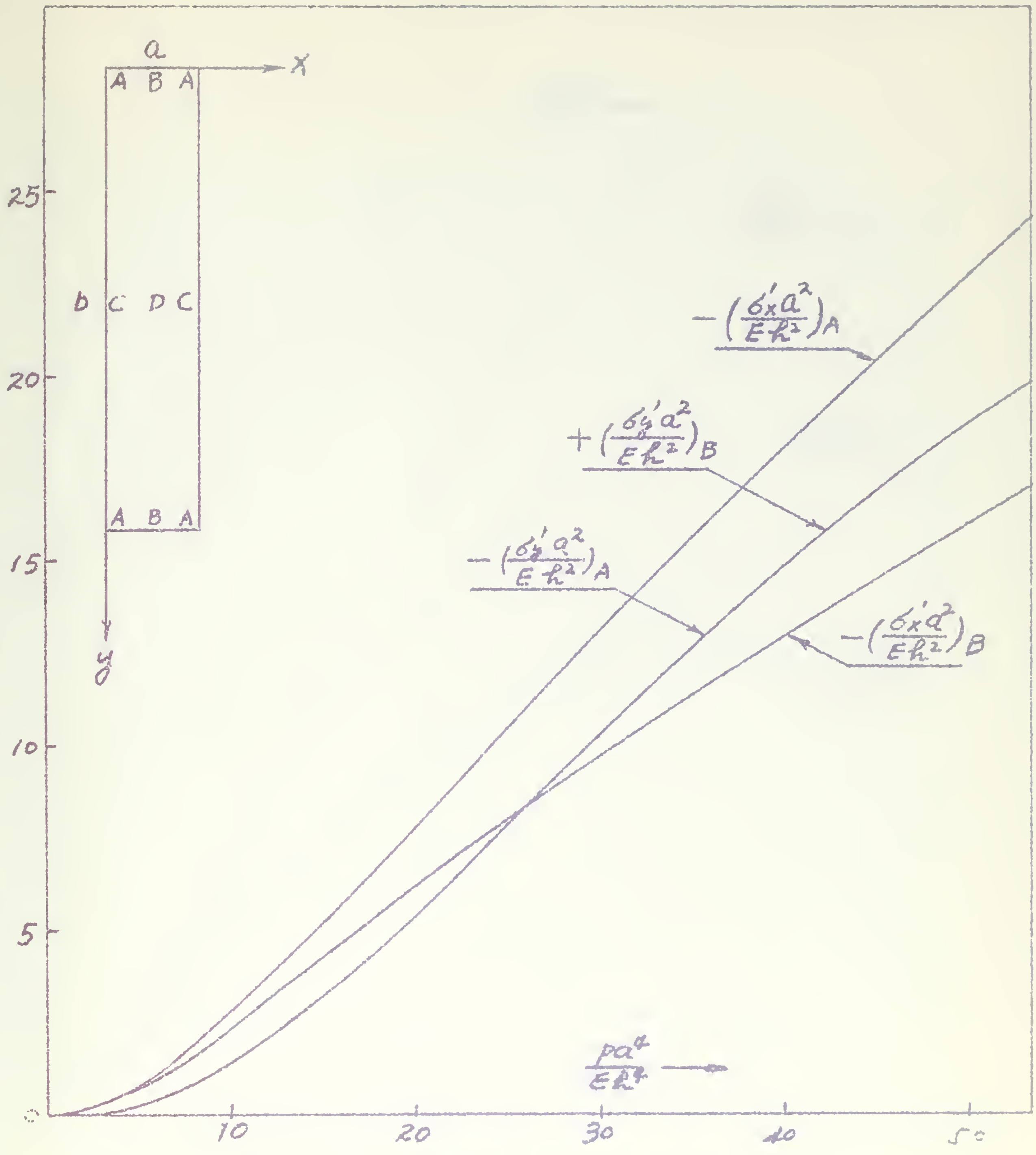


Fig. 8.— Membrane stesses curve

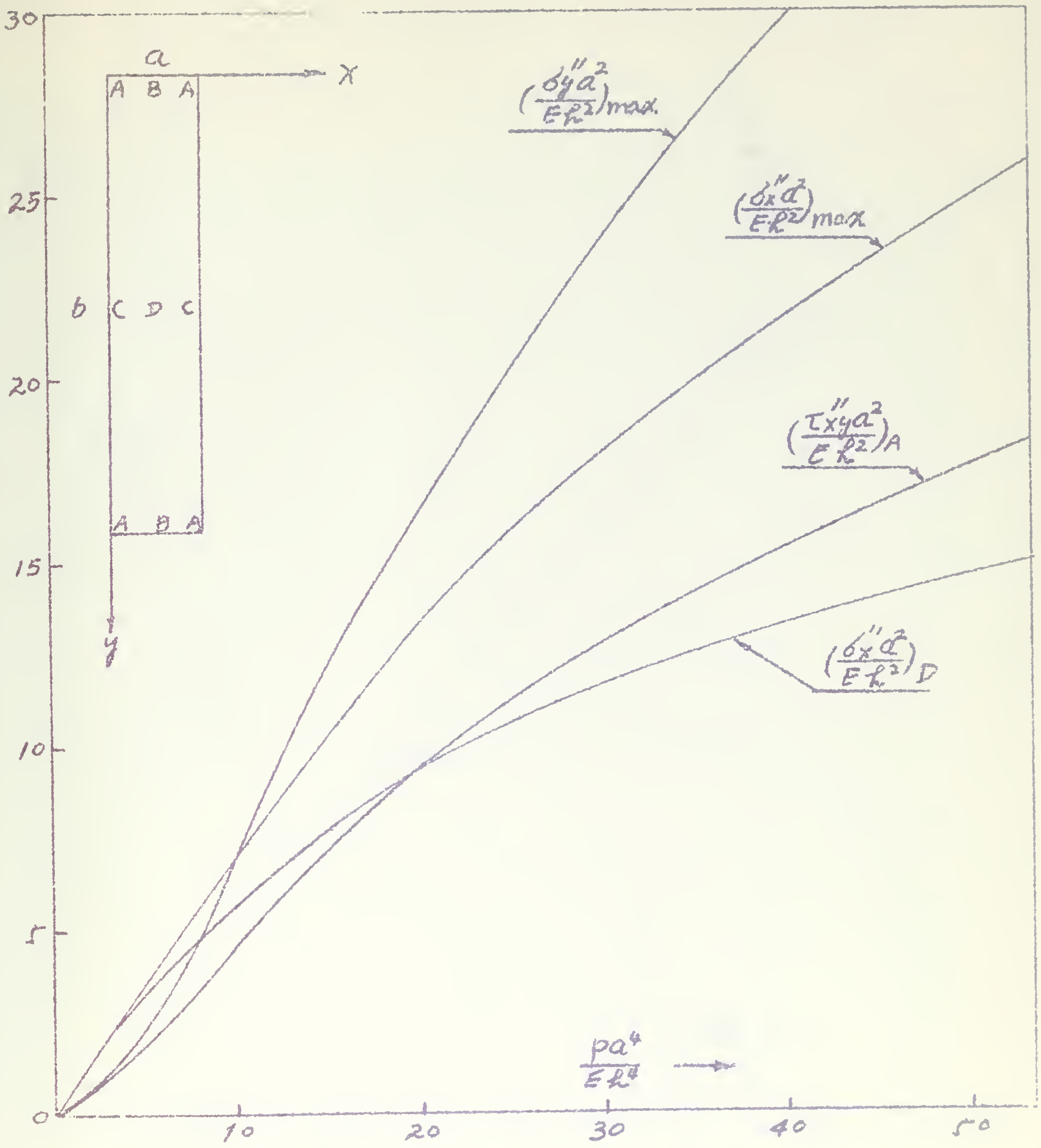


Fig. 9.-- Bending stresses curve

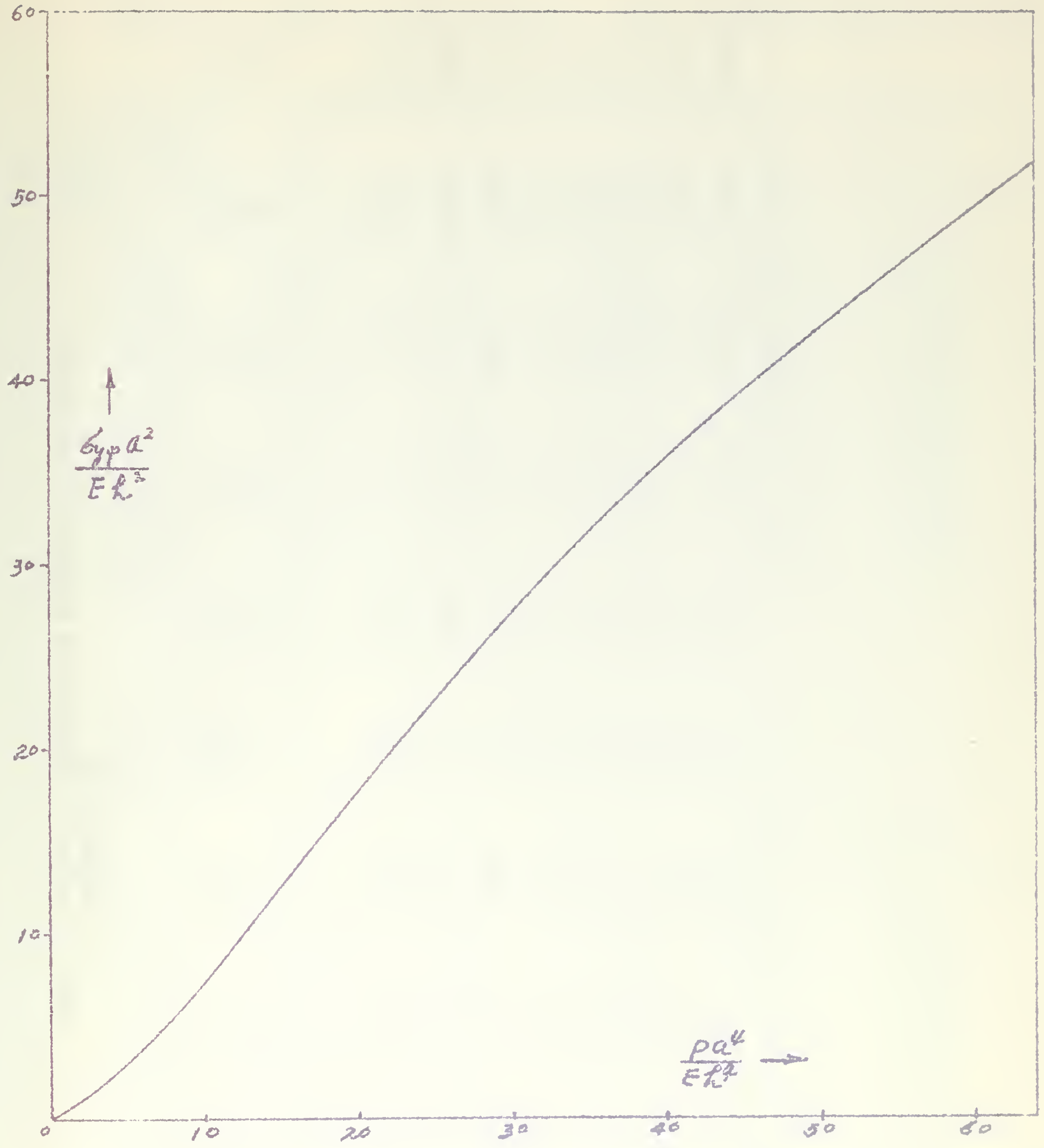


Fig. 10.-- Stress intensity curve

Table 1. Values of coefficients in deflection function, equation (8)

$\beta = 5.0$  $\mu = 0.316$

| $\frac{pa^4}{Eh^4}$ | $\frac{w_{1,1}}{h}$ | $\frac{w_{1,3}}{h}$ | $\frac{w_{1,5}}{h}$ | $\frac{w_{1,7}}{h}$ | $\frac{w_{1,9}}{h}$ | $\frac{w_{1,11}}{h}$ | $\frac{w_{1,13}}{h}$ | $\frac{w_{1,15}}{h}$ | $\frac{w_{center}}{h}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 1.218 | 0.2000 | 0.04222 | 0.01235 | 0.00401 | 0.00147 | 0.00062 | 0.00029 | 0.00015 | 0.16711 |
| 1.851 | 0.3000 | 0.06824 | 0.02134 | 0.00703 | 0.00253 | 0.00102 | 0.00047 | 0.00024 | 0.24781 |
| 3.186 | 0.50000 | 0.13099 | 0.04857 | 0.01772 | 0.00642 | 0.00245 | 0.00102 | 0.00048 | 0.40437 |
| 6.956 | 1.00000 | 0.31142 | 0.15480 | 0.07726 | 0.03596 | 0.01580 | 0.00677 | 0.00292 | 0.79013 |
| 11.510 | 1.50000 | 0.48842 | 0.26946 | 0.15760 | 0.08799 | 0.04614 | 0.02311 | 0.01133 | 1.17707 |
| 17.227 | 2.00000 | 0.65942 | 0.37739 | 0.23719 | 0.14662 | 0.08635 | 0.04867 | 0.02675 | 1.56297 |
| 24.561 | 2.50000 | 0.82744 | 0.48022 | 0.31236 | 0.20465 | 0.13022 | 0.08017 | 0.04838 | 1.94664 |
| 33.980 | 3.00000 | 0.99399 | 0.58023 | 0.38398 | 0.26033 | 0.17453 | 0.11495 | 0.07508 | 2.32793 |
| 45.962 | 3.50000 | 1.15980 | 0.67867 | 0.45328 | 0.31372 | 0.21792 | 0.15121 | 0.10573 | 2.70687 |
| 60.984 | 4.00000 | 1.32510 | 0.77627 | 0.52114 | 0.36533 | 0.26004 | 0.18783 | 0.13925 | 3.08390 |

Table 2. Values of deflection profile, $\frac{w}{h}$, along $x = \frac{a}{2}$

$\beta = 5.0$ $\mu = 0.316$

| $\frac{w}{h}$ $\frac{pa^4}{Eh^4}$ $y/b$ | 1.1218 | 1.851 | 3.186 | 6.956 | 11.510 |
|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.025 | 0.0342 | 0.0543 | 0.1050 | 0.2946 | 0.5396 |
| 0.050 | 0.0652 | 0.1034 | 0.1978 | 0.5353 | 0.9484 |
| 0.075 | 0.0911 | 0.1437 | 0.2710 | 0.6941 | 1.1800 |
| 0.100 | 0.1114 | 0.1748 | 0.3233 | 0.7761 | 1.2385 |
| 0.150 | 0.1388 | 0.2145 | 0.3802 | 0.8138 | 1.2142 |
| 0.200 | 0.1537 | 0.2341 | 0.4003 | 0.8051 | 1.1937 |
| 0.350 | 0.1609 | 0.2423 | 0.4045 | 0.7930 | 1.1745 |
| 0.300 | 0.1645 | 0.2459 | 0.4053 | 0.7915 | 1.1817 |
| 0.350 | 0.1661 | 0.2472 | 0.4048 | 0.7903 | 1.1791 |
| 0.400 | 0.1668 | 0.2476 | 0.4046 | 0.7903 | 1.1787 |
| 0.450 | 0.1671 | 0.2479 | 0.4047 | 0.7907 | 1.1818 |
| 0.500 | 0.1671 | 0.2478 | 0.4044 | 0.7901 | 1.1771 |

| | 17.227 | 24.561 | 33.980 | 45.962 | 69.984 |
|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.025 | 0.8063 | 1.0813 | 1.3593 | 1.6380 | 1.9163 |
| 0.050 | 1.3828 | 1.8210 | 2.2580 | 2.6924 | 3.1241 |
| 0.075 | 1.6440 | 2.1064 | 2.5582 | 3.0019 | 3.4399 |
| 0.100 | 1.6733 | 2.0846 | 2.4805 | 2.8668 | 3.2473 |
| 0.150 | 1.5928 | 1.9662 | 2.3416 | 2.7209 | 3.1040 |
| 0.200 | 1.5880 | 1.9918 | 2.4024 | 2.7814 | 3.2125 |
| 0.250 | 1.5673 | 1.9383 | 2.3655 | 2.6988 | 3.1589 |
| 0.300 | 1.5712 | 1.9784 | 2.3716 | 2.7679 | 3.1967 |
| 0.350 | 1.5697 | 1.9600 | 2.3510 | 2.7381 | 3.1257 |
| 0.400 | 1.5680 | 1.9570 | 2.3454 | 2.7334 | 3.1211 |
| 0.450 | 1.5703 | 1.9601 | 2.3323 | 2.7429 | 3.1934 |
| 0.500 | 1.5630 | 1.9466 | 2.3279 | 2.7069 | 3.0839 |

Table 3. Values of membrane stress, $\frac{\sigma_x' a^2}{E h^2}$, along $y=0, b$

$\beta = 5.0$ $\mu = 0.316$

| $\frac{\sigma_x' a^2}{E h^2}$ $\frac{pa^4}{Eh^4}$ $x/a$ | 1.1218 | 1.851 | 3.186 | 6.956 | 11.510 |
|---|---|---|---|---|---|
| 0.0 | -0.0522 | -0.1186 | -0.3382 | -1.4528 | -3.4578 |
| 0.05 | -0.0522 | -0.1186 | -0.3379 | -1.4498 | -3.4453 |
| 0.10 | -0.0521 | -0.1184 | -0.3372 | -1.4412 | -3.4092 |
| 0.15 | -0.0520 | -0.1181 | -0.3360 | -1.4278 | -3.3529 |
| 0.20 | -0.0519 | -0.1178 | -0.3346 | -1.4109 | -3.2820 |
| 0.25 | -0.0518 | -0.1174 | -0.3330 | -1.3922 | -3.2033 |
| 0.30 | -0.0516 | -0.1170 | -0.3313 | -1.3735 | -3.1247 |
| 0.35 | -0.0515 | -0.1167 | -0.3299 | -1.3567 | -3.0538 |
| 0.40 | -0.0514 | -0.1164 | -0.3287 | -1.3433 | -2.9975 |
| 0.45 | -0.0513 | -0.1162 | -0.3280 | -1.3347 | -2.9614 |
| 0.50 | -0.0513 | -0.1162 | -0.3277 | -1.3317 | -2.9489 |

|  | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|
| 0.0 | -6.3977 | -10.2864 | -15.1262 | -20.9154 | -27.6493 |
| 0.05 | -6.3663 | -10.2255 | -15.0253 | -20.6537 | -27.4364 |
| 0.10 | -6.2752 | -10.0490 | -14.7324 | -20.3235 | -26.8186 |
| 0.15 | -6.1332 | -9.7741 | -14.2761 | -19.6378 | -25.8562 |
| 0.20 | -5.9542 | -9.4276 | -13.7012 | -18.7738 | -24.6436 |
| 0.25 | -5.7559 | -9.0436 | -13.0639 | -17.8160 | -23.2994 |
| 0.30 | -5.5575 | -8.6595 | -12.4266 | -16.8583 | -21.9552 |
| 0.35 | -5.3786 | -8.3131 | -11.8517 | -15.9942 | -20.7426 |
| 0.40 | -5.2366 | -8.0381 | -11.3954 | -15.3086 | -19.7803 |
| 0.45 | -5.1454 | -7.8616 | -11.1025 | -14.8683 | -19.1624 |
| 0.50 | -5.1140 | -7.8008 | -11.0015 | -14.7166 | -18.9495 |

Table 4. Values of membrane stress, $\frac{\sigma_x' a^2}{E h^2}$, along $x = 0, a$

$\beta = 5.0 \qquad \mu = 0.316$

| $\frac{\sigma_x' a^2}{E h^2}$ \ $\frac{p a^4}{E h^4}$ <br> $y/b$ | 1.1218 | 1.851 | 3.186 | 6.956 | 11.510 | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | -0.0522 | -0.1186 | -0.3382 | -1.4528 | -3.4578 | -6.3977 | -10.2864 | -15.1262 | -20.9154 | -27.6493 |
| 0.01 | -0.0517 | -0.1174 | -0.3337 | -1.4167 | -3.3338 | -6.1157 | -9.7720 | -14.3049 | -19.7129 | -25.9928 |
| 0.02 | -0.0504 | -0.1139 | -0.3206 | -1.3182 | -2.9816 | -5.3225 | -9.3363 | -12.0244 | -16.3882 | -21.4276 |
| 0.03 | -0.-481 | -0.1083 | -0.2998 | -1.1532 | -2.4556 | -4.1623 | -6.2676 | -8.7759 | -11.6931 | -15.0245 |
| 0.04 | -0.0452 | -0.1010 | -0.2729 | -0.9556 | -1.8320 | -2.8305 | -3.9498 | -5.2024 | -6.6010 | -8.1566 |
| 0.06 | -0.0379 | -0.9826 | -0.2076 | -0.5245 | -0.6016 | -0.3991 | 0.0384 | 0.6714 | 1.4736 | 2.2484 |
| 0.08 | -0.0295 | -0.0622 | -0.1387 | -0.1527 | 0.2590 | 1.0138 | 2.0270 | 3.2348 | 4.6174 | 6.1659 |
| 0.10 | -0.0213 | -0.0424 | -0.0765 | 0.0967 | 0.6542 | 1.4206 | 2.2980 | 3.2555 | 4.2887 | 5.4029 |
| 0.15 | -0.0041 | -0.0035 | 0.0260 | 0.2820 | 0.6033 | 0.9014 | 1.2049 | 1.5511 | 1.9658 | 2.4624 |
| 0.20 | 0.0067 | 0.0183 | 0.0651 | 0.2325 | 0.3717 | 0.5417 | 0.8043 | 1.1827 | 1.6789 | 2.2857 |
| 0.25 | 0.0123 | 0.0281 | 0.0733 | 0.1736 | 0.2372 | 0.3112 | 0.4002 | 0.4903 | 0.5753 | 0.6553 |
| 0.50 | 0.0172 | 0.0343 | 0.0707 | 0.1499 | 0.2267 | 0.3104 | 0.3993 | 0.4894 | 0.5749 | 0.6541 |

Table 5. Values of membrane stress, $\frac{\sigma_x' a^2}{E h^2}$, along $x = \frac{a}{2}$

$\beta = 5.0 \qquad \mu = 0.316$

| $\frac{\sigma_x a^2}{E h^2}$<br>$\frac{p a^4}{E h^4}$<br>$y/b$ | 1.1218 | 1.851 | 3.186 | 6.956 | 11.510 | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | -0.0513 | -0.1162 | -0.3277 | -1.3317 | -2.9489 | -5.1140 | -7.8008 | -11.0015 | -14.7166 | -18.9495 |
| 0.01 | -0.0508 | -0.1150 | -0.3232 | -1.2953 | -2.8250 | -4.8347 | -7.2950 | -10.1982 | -13.5456 | -17.3420 |
| 0.02 | -0.0494 | -0.1114 | -0.3100 | -1.1909 | -2.4756 | -4.0561 | -5.8975 | -70.9936 | -10.3488 | -12.9715 |
| 0.03 | -0.0472 | -0.1058 | -0.2891 | -1.0321 | -1.9611 | -2.9387 | -3.9303 | -4.9356 | -5.9650 | -7.0324 |
| 0.04 | -0.0443 | -0.0985 | -0.2622 | -0.8381 | -1.3657 | -1.6991 | -1.8183 | -1.7361 | -1.4722 | -1.0456 |
| 0.06 | -0.0370 | -0.0804 | -0.1980 | -0.4294 | -0.2668 | 0.3423 | 1.3397 | 2.6716 | 4.3001 | 6.2020 |
| 0.08 | -0.0289 | -0.0606 | -0.1318 | -0.1023 | 0.3659 | 1.1283 | 2.0403 | 3.0239 | 4.0426 | 5.0852 |
| 0.10 | -0.0210 | -0.0417 | -0.0736 | 0.0911 | 0.5089 | 0.8847 | 1.0602 | 1.7151 | 2.3103 | 2.8012 |
| 0.15 | -0.0044 | -0.0042 | 0.0215 | 0.2020 | 0.2648 | 0.5058 | 0.8138 | 1.4307 | 1.9915 | 2.7303 |
| 0.20 | 0.0063 | 0.0173 | 0.0596 | 0.1815 | 0.2531 | 0.3908 | 0.6868 | 1.1709 | 1.8404 | 2.6795 |
| 0.25 | 0.0119 | 0.0269 | 0.0683 | 0.1470 | 0.2067 | 0.2353 | 0.2403 | 0.2485 | 0.2491 | 0.2533 |
| 0.50 | 0.0171 | 0.0340 | 0.0703 | 0.1464 | 0.2037 | 0.2329 | 0.2385 | 0.2401 | 0.2432 | 0.2453 |

Table 6. Values of membrane stress, $\frac{\sigma_y' a^2}{E h^2}$, along $y = 0, b$

$\beta = 5.0$ $\mu = 0.316$

| $\frac{\sigma_y' a^2}{E h^2}$ $\frac{pa^4}{Eh^4}$ $x/a$ | 1.218 | 1.851 | 3.186 | 6.956 | 11.510 | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | -0.0068 | -0.0177 | -0.0698 | -0.5756 | -1.9288 | -4.2835 | -7.6673 | -12.0739 | -17.4872 | -23.8888 |
| 0.05 | -0.0065 | -0.0169 | -0.0664 | -0.5474 | -1.8344 | -4.0738 | -7.2921 | -11.4830 | -16.6313 | -22.7196 |
| 0.10 | -0.0055 | -0.0144 | -0.0565 | -0.4656 | -1.5605 | -3.4654 | -6.2030 | -9.7680 | -14.1478 | -19.3264 |
| 0.15 | -0.0040 | -0.0104 | -0.0410 | -0.3383 | -1.1337 | -2.5178 | -4.5067 | -7.0969 | -10.2787 | -14.0415 |
| 0.20 | -0.0021 | -0.0055 | -0.0216 | -0.1779 | -0.5960 | -1.3237 | -2.3693 | -3.7311 | -5.4039 | -7.3821 |
| 0.25 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.30 | 0.0021 | 0.0055 | 0.0216 | 0.1779 | 0.5960 | 1.3237 | 2.3693 | 3.7311 | 5.4039 | 7.3821 |
| 0.35 | 0.0040 | 0.0104 | 0.0410 | 0.3383 | 1.1337 | 2.5178 | 4.5067 | 7.0969 | 10.2787 | 14.0415 |
| 0.40 | 0.0055 | 0.0144 | 0.0565 | 0.4656 | 1.5615 | 3.4654 | 6.2030 | 9.7680 | 14.1478 | 19.3264 |
| 0.45 | 0.0065 | 0.0169 | 0.0664 | 0.5474 | 1.8344 | 4.0738 | 7.2921 | 11.4830 | 16.6313 | 22.7196 |
| 0.50 | 0.0068 | 0.0177 | 0.0698 | 0.5756 | 1.9288 | 4.2835 | 7.6673 | 12.0739 | 17.4872 | 23.8888 |

Table 7. Values of membrane stress, $\frac{\sigma_y' a^2}{E h^2}$, along $x = 0, a$

$\beta = 5.0$ $\mu = 0.316$

| $\frac{\sigma_y' a^2}{E h^2}$ $\frac{pa^4}{Eh^4}$ $y/b$ | 1.218 | 1.851 | 3.186 | 6.956 | 11.510 | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | -0.0068 | -0.0177 | -0.0698 | -0.5756 | -1.9288 | -4.2835 | -7.6673 | -12.0739 | -17.4872 | -23.8888 |
| 0.01 | -0.0068 | -0.0177 | -0.0696 | -0.5726 | -1.9163 | -4.2518 | -7.6060 | -11.9722 | -17.3344 | -23.6743 |
| 0.02 | -0.0067 | -0.0175 | -0.0688 | -0.5636 | -1.8786 | -4.1570 | -7.4226 | -11.6681 | -16.8776 | -23.0335 |
| 0.03 | -0.0066 | -0.0172 | -0.0675 | -0.5486 | -1.8160 | -3.9998 | -7.1191 | -11.1655 | -16.1235 | -21.9765 |
| 0.04 | -0.0064 | -0.0168 | -0.0657 | -0.5276 | -1.7291 | -3.7824 | -6.7007 | -10.4742 | -15.0880 | -20.5269 |
| 0.06 | -0.0060 | -0.0155 | -0.0605 | -0.4688 | -1.4891 | -3.1888 | -5.5669 | -8.6115 | -12.3098 | -16.6507 |
| 0.08 | -0.0054 | -0.0139 | -0.0534 | -0.3916 | -1.1847 | -2.4530 | -4.1842 | -6.3669 | -8.9928 | -12.0561 |
| 0.10 | -0.0046 | -0.0119 | -0.0450 | -0.3046 | -0.8595 | -1.6957 | -2.7983 | -4.1616 | -5.7840 | -7.6655 |
| 0.15 | -0.0026 | -0.0067 | -0.0230 | -0.1101 | -0.2331 | -0.3859 | -0.5860 | -0.8523 | -1.1983 | -1.6314 |
| 0.20 | -0.0009 | -0.0022 | -0.0057 | -0.0123 | -0.0391 | -0.0587 | -0.0921 | -0.1350 | -0.2036 | -0.2373 |
| 0.25 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.50 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

Table 8. Values of shearing stress, $\frac{\sigma_{xy}'' a^2}{E h^2}$, along $y = 0$

$\beta = 5.0$ $\mu = 0.316$

| $\frac{\sigma_{xy}'' a^2}{E h^2}$ \ $\frac{p a^4}{E h^4}$ <br> $x/a$ | 1.218 | 1.851 | 3.186 | 6.956 | 11.510 |
|---|---|---|---|---|---|
| 0.0 | -0.3319 | -0.5282 | -1.0237 | -2.9086 | -5.3890 |
| 0.5 | -0.2734 | -0.5217 | -1.0111 | -2.8728 | -5.3226 |
| 0.10 | -0.1702 | -0.5024 | -0.9736 | -2.7663 | -5.1252 |
| 0.15 | -0.0968 | -0.4707 | -0.9122 | -2.5916 | -4.8016 |
| 0.20 | -0.0488 | -0.4273 | -0.8282 | -2.3531 | -4.3598 |
| 0.25 | -0.0236 | -0.3735 | -0.7239 | -2.0567 | -3.8106 |
| 0.30 | -0.0117 | -0.3105 | -0.6017 | -1.7096 | -3.1676 |
| 0.35 | -0.0044 | -0.2398 | -0.4648 | -1.3205 | -2.4465 |
| 0.40 | -0.0028 | -0.1632 | -0.3164 | -0.8988 | -1.6653 |
| 0.45 | -0.0004 | -0.0826 | -0.1602 | -0.4550 | -0.8430 |
| 0.50 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

|  | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|
| 0.0 | -8.1214 | -10.9586 | -13.8402 | -16.7378 | -19.6369 |
| 0.05 | -8.0214 | -10.8237 | -13.6698 | -16.5317 | -19.3951 |
| 0.10 | -7.7239 | -10.4223 | -13.1628 | -15.9186 | -18.6758 |
| 0.15 | -7.2362 | -9.7642 | -12.3317 | -14.9138 | -17.4966 |
| 0.20 | -6.5703 | -8.8657 | -11.1969 | -13.5412 | -15.8866 |
| 0.25 | -5.7427 | -7.7486 | -9.7865 | -11.8354 | -13.8854 |
| 0.30 | -4.7736 | -6.4413 | -8.1351 | -9.8382 | -11.5423 |
| 0.35 | -3.6870 | -4.9751 | -6.2833 | -7.5988 | -8.9150 |
| 0.40 | -2.5096 | -3.3864 | -4.2769 | -5.1723 | -6.0681 |
| 0.45 | -1.2705 | -1.7143 | -2.1651 | -2.6184 | -3.0719 |
| 0.50 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

Table 9. Values of shearing stress, $\frac{\sigma_{xy}'' a^2}{E h^2}$, along $x = 0$

$\beta = 5.0 \qquad \mu = 0.316$

| $\frac{\sigma_{xy}'' a^2}{E h^2}$ \ $\frac{p a^4}{E h^4}$ <br> $y/b$ | 1.218 | 1.851 | 3.186 | 6.956 | 11.510 | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | -0.3319 | -0.5282 | -1.0237 | -2.9086 | -5.3890 | -8.1214 | -10.9586 | -13.8402 | -16.7378 | -19.6369 |
| 0.01 | -0.3292 | -0.5237 | -1.0134 | -2.8619 | -5.2727 | -7.9134 | -10.6458 | -13.4147 | -16.1950 | -18.9743 |
| 0.02 | -0.3213 | -0.5105 | -0.9833 | -2.7257 | -4.9357 | -7.3131 | -9.7453 | -12.1921 | -14.6378 | -17.0757 |
| 0.03 | -0.3088 | -0.4894 | -0.9353 | -2.5115 | -4.4121 | -6.3877 | -8.3648 | -10.3256 | -12.2678 | -14.1930 |
| 0.04 | -0.2925 | -0.4520 | -0.8726 | -2.2371 | -3.7533 | -5.2382 | -6.6653 | -8.0433 | -9.3844 | -10.6999 |
| 0.05 | -0.2734 | -0.4297 | -0.7993 | -1.9240 | -3.0213 | -3.9848 | -4.8367 | -5.6121 | -6.3372 | -7.0308 |
| 0.06 | -0.2525 | -0.3946 | -0.7194 | -1.5950 | -2.2799 | -2.7485 | -3.0693 | -3.2978 | -3.4707 | -3.6120 |
| 0.07 | -0.2310 | -0.3582 | -0.6372 | -1.2718 | -1.5872 | -1.6374 | -1.5273 | -1.3256 | -1.0737 | -0.7966 |
| 0.08 | -0.2096 | -0.3222 | -0.5562 | -0.9726 | -0.9888 | -0.7309 | -0.3272 | -0.0981 | -0.0501 | - |
| 0.09 | -0.1892 | -0.2877 | -0.4795 | -0.7105 | -0.5130 | -0.0728 | - | - | - | - |
| 0.10 | -0.1702 | -0.2556 | -0.4089 | -0.4931 | -0.1702 | - | - | - | - | - |

Table 10. Values of bending stress, $\frac{\sigma_x'' a^2}{E h^2}$ , along $x = \frac{a}{2}$

$\beta = 5.0$ $\mu = 0.316$

| $\frac{\sigma_x'' a^2}{E h^2}$ $\frac{pa^4}{Eh^4}$ $y/b$ | 1.218 | 1.851 | 3.186 | 6.956 | 11.510 |
|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.02 | 0.1812 | 0.2909 | 0.5803 | 1.8304 | 3.6925 |
| 0.04 | 0.3451 | 0.5530 | 1.0946 | 3.3322 | 6.4987 |
| 0.05 | 0.4168 | 0.6667 | 1.3122 | 3.8909 | 7.3981 |
| 0.06 | 0.4807 | 0.7674 | 1.4995 | 4.3087 | 7.9438 |
| 0.07 | 0.5368 | 0.8550 | 1.6566 | 4.5918 | 8.1689 |
| 0.08 | 0.5856 | 0.9301 | 1.7851 | 4.7563 | 8.1349 |
| 0.10 | 0.6649 | 1.0491 | 1.9700 | 4.8262 | 7.6069 |
| 0.15 | 0.7955 | 1.2316 | 2.1843 | 4.5562 | 6.5496 |
| 0.20 | 0.8643 | 1.3156 | 2.2377 | 4.4249 | 6.5015 |
| 0.50 | 0.9157 | 1.3575 | 2.2141 | 4.3164 | 6.3860 |

| | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.02 | 5.8915 | 8.2681 | 10.7408 | 13.2646 | 15.8121 |
| 0.04 | 10.1177 | 13.9483 | 17.8796 | 21.8558 | 25.8460 |
| 0.05 | 11.3017 | 15.3631 | 19.4842 | 23.6203 | 27.7513 |
| 0.06 | 11.8529 | 15.8279 | 19.7995 | 23.7452 | 27.6595 |
| 0.07 | 11.8525 | 15.4884 | 19.0483 | 22.5370 | 25.9682 |
| 0.08 | 11.4352 | 14.5736 | 17.5678 | 20.4515 | 23.2568 |
| 0.10 | 10.0083 | 12.0951 | 13.9664 | 15.6983 | 17.3451 |
| 0.15 | 8.4660 | 10.4010 | 12.3814 | 14.1381 | 16.0569 |
| 0.20 | 8.4077 | 10.3521 | 12.2315 | 14.0615 | 15.8451 |
| 0.50 | 8.3975 | 10.3435 | 12.2269 | 14.0587 | 15.8340 |

Table 11. Values of bending stress, $\frac{6y''a^2}{Eh^2}$, along $x = \frac{a}{2}$

$\beta = 5.0$ $\mu = 0.316$

| $\frac{6y''a^2}{Eh^2}$ $\frac{pa^4}{Eh^4}$ $y/b$ | 1.218 | 1.851 | 3.186 | 6.956 | 11.510 |
|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.01 | 0.0737 | 0.1218 | 0.2646 | 1.0740 | 2.5349 |
| 0.02 | 0.1436 | 0.2371 | 0.5146 | 2.0671 | 4.8339 |
| 0.03 | 0.2063 | 0.3404 | 0.7369 | 2.9083 | 6.6937 |
| 0.04 | 0.2591 | 0.4274 | 0.9216 | 3.5462 | 7.9706 |
| 0.05 | 0.3008 | 0.4956 | 1.0626 | 3.9515 | 8.5982 |
| 0.06 | 0.3309 | 0.5444 | 1.1585 | 4.1224 | 8.5922 |
| 0.07 | 0.3503 | 0.5752 | 1.2117 | 4.0811 | 8.0433 |
| 0.08 | 0.3607 | 0.5905 | 1.2282 | 3.8691 | 7.0981 |
| 0.09 | 0.3644 | 0.5941 | 1.2161 | 3.5401 | 5.9335 |
| 0.10 | 0.3635 | 0.5899 | 1.1845 | 3.1511 | 4.7279 |

|  | 17.227 | 24.561 | 33.980 | 45.962 | 60.984 |
|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.01 | 4.4191 | 6.5519 | 8.8308 | 11.1945 | 13.6034 |
| 0.02 | 8.3748 | 12.3631 | 16.6109 | 21.0069 | 25.4803 |
| 0.03 | 11.4717 | 16.8066 | 22.4553 | 28.2779 | 34.1872 |
| 0.04 | 13.4392 | 19.4623 | 25.7814 | 32.2538 | 38.7947 |
| 0.05 | 14.1667 | 20.1754 | 26.3912 | 32.6961 | 39.0261 |
| 0.06 | 13.7127 | 19.0680 | 24.4876 | 29.9004 | 34.2782 |
| 0.07 | 12.2857 | 16.5058 | 20.6213 | 24.6224 | 28.5243 |
| 0.08 | 10.2031 | 13.0251 | 15.5815 | 17.9272 | 20.1202 |
| 0.09 | 7.8337 | 9.2353 | 10.2511 | 10.9923 | 11.5511 |
| 0.10 | 5.5366 | 5.7158 | 5.4544 | 4.9020 | 4.1700 |

## REFERENCES

1. S. Levy. "Bending of Rectangular Plates With Large Deflections," National Advisory Committee for Aeronautics, Technical Note 846, 1942.

2. H.A. Schade. "Class Notes of Theory of Ship Structure."

3. S. Timoshenko. Theory of Plates and Shells.

4. A guide for the analysis of ship structures.

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